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Discussed with Ai Yifei

No outside material consulted

Proof for Question No.1:

Suppose there exist a function C(Q), that, given input Q, outputs:

T, if Q is a syntactically correct Python program, there is some input on which Q halts, and there is some input on which Q does not halt.

F, otherwise.

def HHH(M):

if C(HHH):

while True:

pass

elif M == “1”:

while True:

pass

elif M == “2”:

return 240

If C(HHH) = T: For HHH(M), whatever M is, HHH(M) won’t halt. So, C(HHH) should be F.

If C(HHH) = F: For HHH(M), If M = “2”, HHH(M) halts. If M == “1”, HHH(M) won’t halt. There is some input on which Q halts, and there is some input on which Q does not halt. So, C(HHH) should be T. (reduction part)

Then this HHH(M) is a contradiction. Thus this kind of C(Q) does not exist.

Proof for Question No.2:

2.(a)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | T |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | T |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | T |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | T |
| 1 | 1 | 0 | 0 | 1 | 2 | 2 | T |
| 1 | 0 | 1 | 0 | 1 | 2 | 2 | T |
| 0 | 1 | 1 | 0 | 1 | 2 | 2 | T |
| 1 | 1 | 1 | 1 | 1 | 3 | 3 | T |

2.(b)

Prove by induction:

1.Suppose for all

Base case:

2.for n = 0, , ,

3. = = = (directly prove from 1 and 2)

Induction step:

3. let be arbitrary,

5. Assume

6. ,, (by definition of binary representation of integers)

7. +=

(directly prove from 6)

=

(directly prove from 5)

=

(directly prove from 1)

= + (by definition of binary representation of integers)

8.

++ (directly prove from 7)

9..

++ (by generalization)

10.. (by induction)